Classifier fusion and imperfect data management

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Bruxelles, January 28, 2020

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Outline

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1. Introduction to information fusion
2. Theory of belief functions for classifier fusion
3. Managing conflict
4. Decisions with conflicting bbas

What is information?

: This shop proposes a lot of beers.

What is information?

: This shop proposes a lot of beers.

Imprecise proposition

What is information?

: This shop proposes a lot of beers.

Imprecise proposition

: This shop proposes 68 different kinds of beers.

What is information?

: This shop proposes a lot of beers.

Imprecise proposition

: This shop proposes 68 different kinds of beers.

Precise proposition

What is information?

: This shop proposes a lot of beers.

Imprecise proposition

: This shop proposes 68 different kinds of beers.

Precise proposition

Imprecision is a kind of imperfection of information

What is information?

: There are some Belgian beers.

What is information?

 : There are some Belgian beers.

Bad condition

Uncertain proposition

What is information?

 : There are some Belgian beers.

Bad condition

Uncertain proposition

Good condition

Certain proposition

What is information?

 : There are some Belgian beers.

Bad condition
Uncertain proposition

Good condition
Certain proposition

Uncertainty is another kind of imperfection of information

How are the sources?

: I am sure that is a Pils.

A

: Maybe it's a Belgian beer.

B

FUSION

How are the sources?

: I am sure that is a Pils.

A

: Maybe it's a Belgian beer.

B

Conflict of information sources has to be solved

FUSION

How are the sources?

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FUSION

A is often mistaken

B is never wrong

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FUSION Reliability of information sources has to be considered

A is often mistaken

B is never wrong

Interest of information fusion

Assume all the sources are completly reliable giving totally certain and precise information

FUSION

Interest of information fusion

Assume all the sources are completly reliable giving totally certain and precise information

 : This is a Chimay beer

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FUSION

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Imperfections

Imperfections on information

- Uncertainty: Degree of conformity to the reality
- Imprecision: quantitative default of knowledge on the information contain
- Incompleteness: lack of information

Sources of information

- Reliability: to give an indisputable information
- Independence: Sources are not linked (assumption often made)
- Conflicting sources: Sources give information in contradiction

Information fusion

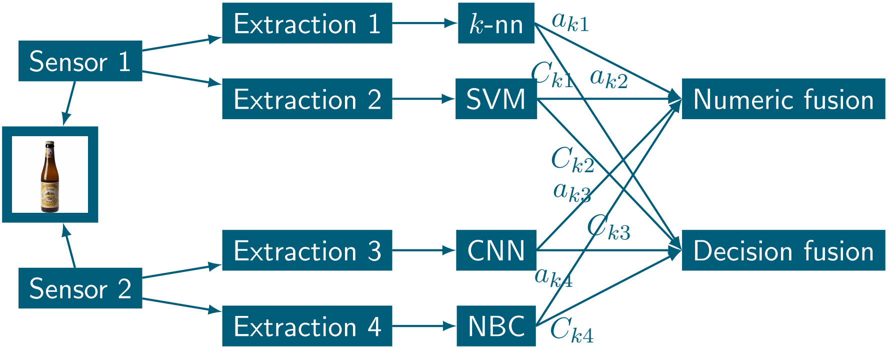
Goal: To combine information coming from many imperfect sources in order to improve the decision making taking into account of imprecisions and uncertainties
To combine information coming from imperfect sources leads fatally to conflict

Three actions are possible face to imperfections:

1. We can try to suppress them
2. We can tolerate them and so we need robust algorithms against these imperfections
3. We can model them

To model imperfections: uncertainty theories:
Probability theory (Bayesian approach) or possibility theory or the theory of belief functions

Classifier fusion



Fusion architecture for classifiers fusion

s sources S_1, S_2, ..., S_s that must take a decision on an observation x in a set of n classes $x \in \Omega = \{\omega_1, \omega_2, \ldots, \omega_n\}$ classes

$$\begin{array}{cc} & \begin{array}{ccccc} \omega_1 & \ldots & \omega_i & \ldots & \omega_n \end{array} \\ \begin{array}{c} S_1 \\ \vdots \\ S_j \\ \vdots \\ S_s \end{array} & \left[\begin{array}{ccccc} M_1^1(x) & \ldots & M_i^1(x) & \ldots & M_n^1(x) \\ \vdots & \ddots & \vdots & \ddots & \vdots \\ M_1^j(x) & \ldots & M_i^j(x) & \ldots & M_n^j(x) \\ \vdots & \ddots & \vdots & \ddots & \vdots \\ M_1^s(x) & \ldots & M_i^s(x) & \ldots & M_n^s(x) \end{array} \right] \end{array} \tag{1}$$

Steps of information fusion

Problems

- How to combine the information coming from all the sources?
- How to take the decision?

Under-problems depending on the application

- How to model information? *i.e.* choice of a formalism
- How to estimate the model parameters?

4 steps

1. Modeling
2. Estimation
3. Combination
4. Decision

Voting process

Modeling: Indicator functions

$$M_i^j(x) = \begin{cases} 1 \text{ if } S_j \; : x \in \omega_i \\ 0 \text{ otherwise} \end{cases}$$

Estimation: α_{ij}: reliability of a source for a given class

Combination:

$$M_i^E(x) = \sum_{j=1}^{s} \alpha_{ij} M_i^j(x)$$

Decision:

$x \in \omega_k$ if $M_k^E(x) = \max_i M_i^E(x) \geq c.s + b(x)$
$x \in \omega_{n+1}$ otherwise *i.e.* no decision
$c \in [0, 1]$, $b(x)$ function of $M_k^E(x)$

Voting process: A result

- **Assumptions**
  - Statistical independence of sources
  - Same probability of success p
  - s odd and > 2 (same kind of result if s is even)
- Then
  With P_R the probability of success after fusion by voting process
  - If $p > 0.5$, P_R tends toward 1 with s
  - If $p < 0.5$, P_R tends toward 0 with s
  - If $p = 0.5$, $P_R = 0.5$ for all s

Behind this result: The majority should have right for fusion.

Probability theory for information fusion

Modeling: A probability is a positive and additive measure, p is defined on a σ-algebra of $\Omega = \{\omega_1, \omega_2, \ldots, \omega_n\}$ and takes values in [0,1].
It verifies: $p(\emptyset) = 0$, $p(\Omega) = 1$, $\displaystyle\sum_{X \in \Omega} p(X) = 1$

Estimation: Choice of the distribution, and/or estimation of parameters

Combination: Bayes rule

$$p(x \in \omega_i / S_1, \ldots, S_s) = \frac{p(S_1, \ldots, S_s / x \in \omega_i) p(x \in \omega_i)}{p(S_1, \ldots, S_s)} \qquad (2)$$

Independence assumption most of the time necessary

Decision: *a posteriori* maximum, likelihood maximum, mean maximum, *etc.*

Limits of the theory of probabilities

- Difficulties to model the absence of knowledge (ex: Sirius)
- Constraint on the classes (exhaustive and exclusive)
- Constraint on the measures (additivity)

Example of Smets on the matter of additivity

If one symptom f (for fiver) is always true when a patient get a illness A (coronavirus) ($p(f|A) = 1$), and if we observe this symptom f, then the probability of the patient having A increases (because $p(A|f) = p(A)/p(f)$ so $p(A|f) \geq p(A)$).

The additivity constraint require then that the probability of the patient having not A decreases:
$p(\overline{A}|f) = 1 - p(A|f)$ so $p(\overline{A}|f) \leq p(\overline{A})$

While there is no reason if the symptom f can be also observe in some other diseases.

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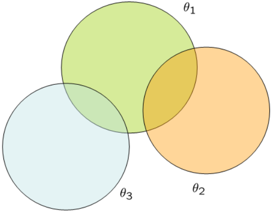
Bases on belief functions

- Use of functions defined on sub-sets instead of singletons such as probabilities
- Discernment frame: $\Omega = \{\omega_1, \ldots, \omega_n\}$, with ω_i are exclusive and exhaustive classes
- Power set: $2^\Omega = \{\emptyset, \{\omega_1\}, \{\omega_2\}, \{\omega_1 \cup \omega_2\}, \ldots, \Omega\}$.
- Several functions in one to one correspondence model uncertainty and imprecision: mass functions, belief functions, plausibility functions
- Extension of 2^Ω to D^Ω, hyper power set in order to model the conflicts
  - D^Ω closed set by union and intersection operators
  - D_r^Ω: reduced set with constraints $(\omega_2 \cap \omega_3 \equiv \emptyset)$

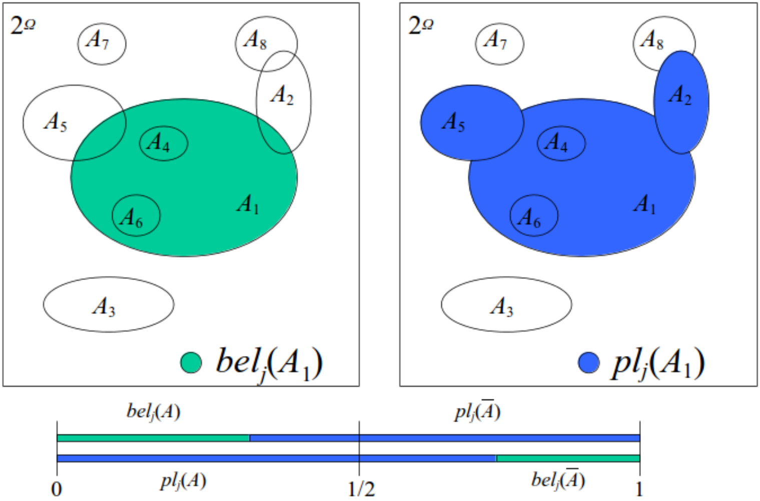


- The basic belief functions (bba or mass functions) are defined on 2^Ω and take values in $[0,1]$
- Normalization condition: $\displaystyle\sum_{X\in 2^\Omega} m(X) = 1$
- A focal element is an element X of 2^Ω such as $m(X) > 0$
- Closed world: $m(\emptyset) = 0$
- We note m_j the mass function of the source S_j

Special cases

- if only focal elements are ω_i then m_j is a **probability**
- $m_j(\Omega) = 1$: total **ignorance** of S_j
- categorical mass function: $m_j(X) = 1$ (noted m_X): S_j has an imprecise knowledge
- $m_j(\omega_i) = 1$: S_j has a precise knowledge
- simple mass functions X^w:
  $m_j(X) = w$ and $m_j(\Omega) = 1 - w$: S_j has an **uncertain and imprecise** knowledge

Belief and plausibility functions



Belief and plausibility functions

- Belief function for $X \in 2^\Omega$:

$$\mathrm{bel}(X) = \sum_{Y \subseteq X} m(Y)$$

  and $\mathrm{bel}(\Omega) = 1 - m(\emptyset)$ (1 in closed world)
- If $m(X) = 0$ does not mean that X is impossible (if $\mathrm{bel}(X) > 0$), but we don't know how to affect a degree of belief precisely on X
- Plausibility function for $X \in 2^\Omega$:

$$\mathrm{pl}(X) = \sum_{Y \cap X \neq \emptyset} m(Y) = \mathrm{bel}(\Omega) - \mathrm{bel}(\overline{X}) \qquad (3)$$

Exercice

Such m a mass function given on $\Omega = \{\omega_1, \omega_2, \omega_3\}$ by:
$m(\omega_1) = 0.2$, $m(\omega_1, \omega_2) = 0.5$, and $m(\omega_1, \omega_2, \omega_3) = 0.3$
Compute $\mathrm{bel}(X)$ and $\mathrm{pl}(X)$ for all $X \in 2^\Omega$.

Discounting

From (Shafer, 1976):

$$m_j^\alpha(X) = \alpha_j m_j(X), \forall X \in 2^\Omega$$
$$m_j^\alpha(\Omega) = 1 - \alpha_j(1 - m_j(\Omega))$$

$\alpha_j \in [0, 1]$ discounting coefficient can be seen as the reliability of the source S_j
If $\alpha_j = 0$ the source are completely unreliable, all the mass is transfered on Ω, the total ignorance

The discounting process increases the intervals $[\mathrm{bel}_j, \mathrm{pl}_j]$ (and so reduces the global conflict in the conjunctive combination rule)

Go back on fusion architecture

s sources S_1, S_2, ..., S_s that must take a decision on an observation x in a set of n classes $x \in \Omega = \{\omega_1, \omega_2, \ldots, \omega_n\}$ classes

$$
\begin{array}{cc}
 & \begin{array}{ccccc} \omega_1 & \ldots & \omega_i & \ldots & \omega_n \end{array} \\
\begin{array}{c} S_1 \\ \vdots \\ S_j \\ \vdots \\ S_s \end{array} &
\left[\begin{array}{ccccc}
m_1^1(x) & \ldots & m_i^1(x) & \ldots & m_n^1(x) \\
\vdots & \ddots & \vdots & \ddots & \vdots \\
m_1^j(x) & \ldots & m_i^j(x) & \ldots & m_n^j(x) \\
\vdots & \ddots & \vdots & \ddots & \vdots \\
m_1^s(x) & \ldots & m_i^s(x) & \ldots & m_n^s(x)
\end{array}\right]
\end{array}
$$

Distance based model (Denœux 1995)

Only ω_i and Ω are focal elements, $n * s$ sources (experts)

- Prototypes case ($\mathbf{x}_i$ center of ω_i). For the observation x

$$
\begin{aligned}
m_j^i(\omega_i) &= \alpha_{ij} \exp[-\gamma_{ij} d^2(x, \mathbf{x}_i)] \\
m_j^i(\Omega) &= 1 - \alpha_{ij} \exp[-\gamma_{ij} d^2(x, \mathbf{x}_i)]
\end{aligned}
$$

  - $0 \leq \alpha_{ij} \leq 1$: discounting coefficient and $\gamma_{ij} > 0$, are parameters to play on the quantity of ignorance and on the form of the mass functions
  - The distance allows to give a mass to x higher according to the proximity to ω_i
- belief k-nn: we consider the k-nearest neighbors instead to $\mathbf{x}_i$
- Then we combine the bbas

- 2 models proposed by Appriou according to three axioms:
  1. the $n * s$ couples $[m_i^j, \alpha_{ij}]$ are distinct information sources where focal elements are: ω_i, $\overline{\omega_i}$ and Ω
  2. If $m_i^j(\omega_i) = 0$ and the information is valid $(\alpha_{ij} = 1)$ then it is certain that ω_i is not true.
  3. Conformity to the Bayesian approach (case where $p(S_j|\omega_j)$ is exactly the reality $(\alpha_{ij} = 1)$ for all i, j) and all the *a priori* probabilities $p(\omega_i)$ are known)
- Need to estimate $p(S_j|\omega_i)$

Probabilistic based model

Model 1:

$$\begin{aligned} m_j^i(\omega_i) &= \frac{\alpha_{ij} R_j p(S_j|\omega_j)}{1 + R_j p(S_j|\omega_j)} \\ m_j^i(\overline{\omega_i}) &= \frac{\alpha_{ij}}{1 + R_j p(S_j|\omega_j)} \\ m_j^i(\Omega) &= 1 - \alpha_{ij} \end{aligned}$$

with $R_j \geq 0$ a normalization factor.

Model 2:

$$\begin{aligned} m_j^i(\omega_i) &= 0 \\ m_j^i(\overline{\omega_i}) &= \alpha_{ij}(1 - R_j p(S_j|\omega_j)) \\ m_j^i(\Omega) &= 1 - \alpha_{ij}(1 - R_j p(S_j|\omega_j) \end{aligned}$$

with $R_j \in [0, (\max_{S_j,i}(p(S_j|\omega_j)))^{-1}]$

Adapted to the cases where we learn on class against the other (SVM)

Probabilistic vs Distance

Difficulties:

- Appriou: learning the probabilities $p(S_j|\omega_j)$
- Denœux: choice of the distance $d(x, \mathbf{x}_i)$

Easiness:

- $p(S_j|\omega_j)$ easier to estimate on decisions with the confusion matrix of the classifiers
- $d(x, \mathbf{x}_i)$ easier to choose on the numeric outputs of classifiers (ex.: Euclidean distance)

Conjunctive rules

- Assume: two cognitively independent and reliable sources S_1 and S_2.
- The conjunctive rule is given for m_1 and m_2 bbas of S_1 and S_2, for all $X \in 2^\Omega$, with $X \neq \emptyset$ by:

$$m_{\text{Conj}}(X) = \sum_{Y_1 \cap Y_2 = X} m_1(Y_1) m_2(Y_2) \tag{4}$$

	$\emptyset$	ω_1	ω_2	ω_3	Ω
m_1	0	0.5	0.1	0	0.4
m_2	0	0.2	0	0.5	0.3
m					

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m_1	0	0.5	0.1	0	0.4
m_2	0	0.2	0	0.5	0.3
m	0.32	0.33	0.03	0.2	0.12

Dempster's rule

- Dempster's rule:

$$m_{\mathrm{D}}(X) = \frac{1}{1-\kappa} m_{\mathrm{Conj}}(X) \tag{5}$$

  where $\kappa = \sum_{A \cap B = \emptyset} m_1(A) m_2(B)$ is generally called conflict or *global conflict*. That is the sum of the *partial conflicts*.
- That is not a conflict measure (Liu, 2006).
- Conjunctive rules are not idempotent

Zadeh example

	$\emptyset$	ω_1	ω_2	ω_3	Ω	Decision
m_1	0	0.9	0	0.1	0	ω_1
m_2	0	0	0.9	0.1	0	ω_2
m_D						

Solutions

- Conflict coming from a false assumption of closed world (PCR6 in DSmT)
- Conflict coming from the assumption of source's independence (average, cautious rule)
- Conflict coming from source's ignorance assumption (Yager, Dubois and Prade)
- Conflict coming from source reliability assumption (disjunctive rule, Florea, Martin and Osswald)
- Conflict coming from a number of sources (LNS rule)

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Decision on Ω

- In general the decision is made on Ω and not on 2^Ω
  - Pessimist: $\max_{\omega\in\Omega} bel(\omega)$
  - Optimist: $\max_{\omega\in\Omega} pl(\omega)$
  - Compromise: $\max_{\omega\in\Omega} betP(\omega)$

Pignistic probability:

$$\mathrm{betP}(\omega) = \sum_{Y\in 2^\Omega, \omega\cap Y\neq\emptyset} \frac{1}{|Y|}\frac{m(Y)}{1-m(\emptyset)} \tag{6}$$

Decision on the unions

- Decision on 2^Ω
- The decision functions f_d (belief, plausibility, pignistic probability, etc.) increase by inclusion (Appriou, 2014)

$$A = \underset{X \in 2^\Omega}{\operatorname{argmax}} \left(m_d(X) f_d(X) \right),$$

where

$$m_d(X) = \left(\frac{K_d \lambda_X}{|X|^r} \right)$$

with $r \in [0, 1]$, is a weighted factor of the wanted precision of the decision:
$r = 1$: singleton,
$r = 0$: ignorance

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Conflict origins

Definition *The* **conflict** *in the theory of belief functions can be defined by the contradiction between two or more mass functions.*

- The sources are not reliable:
  reliability and conflict are very linked in this case
- The discernment frame is not exhaustive:
  the closed world assumption must be avoid
- The sources do not express on the same phenomena:
  bbas must not be combined

Let note $\mathrm{Conf}(m_1, m_2)$ a conflict measure between two mass functions m_1 and m_2.

Confusion in conflict

- Many measures are called conflict in the theory of belief functions but are not conflict
- The global conflict contains an indication of the conflict between bbas, but not only
- Some terms such as internal conflict, discord, contradiction are not conflict
- Some uncertainty measures such as entropic measures can be sometimes called conflict, but are not conflict

Auto-conflict: a discord measure

Introduced by Osswald and Martin, 2006, the auto-conflict of order s for one expert is given by:

$$a_s = \left(\textcircled{\cap}_{j=1}^{s} m\right)(\emptyset). \tag{7}$$

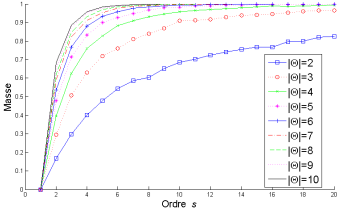
where $\textcircled{\cap}$ is the conjunctive operator.

The following property holds:

$a_s \leq a_{s+1}$

$\lim_{s \to +\infty} a_s = 1$ if the intersection of all focal elements of m is empty

The global conflict must be weighted with the auto-conflict.



Mean of auto-conflict for generated bbas

Conflict measures

- Based on the global conflict coming from the conjunctive rule:

  $\mathrm{Conf}(m_1, m_2) = m_{\mathrm{Conj}}(\emptyset)$ (point of view of Smets (2007))
  $\mathrm{Conf}(m_1, m_2) = -\ln(1 - m_{\mathrm{Conj}}(\emptyset))$ Yager (1983)

  - pros: $\mathrm{Conf}(m_1, m_\Omega) = 0$
    where $m_\Omega(\Omega) = 1$ is the total ignorance
  - cons:
    1. $\mathrm{Conf}(m_1, m_1) \neq 0$ because of the non-idempotence
    2. $m_{\mathrm{Conj}}(\emptyset)$ contains a part of the auto-conflict of m_1 and m_2
    3. The conjunctive rule needs the independence assumption between the sources: There is no reason to get a link between independence and conflict

Conflict measure: based on distance

$\mathrm{Conf}(m_1, m_2) = d(m_1, m_2)$ (Martin *et al.*, 2008)
Conflict between an expert S_j and the $s-1$ other experts is given by the the mean of conflicts two by two:

$$\mathrm{Conf}(m_j, m_{\mathcal{E}}) = \frac{1}{s-1} \sum_{e=1, e\neq j}^{s} \mathrm{Conf}(j, e) \tag{8}$$

Another definition is given by: $\mathrm{Conf}(m_j, m_s) = d(m_j, m_{\mathcal{E}})$
where $m_{\mathcal{E}}$ is the bba of the artificial expert build by the combined bbas of the $s-1$ other experts of $\mathcal{E}$ without the expert S_j.

- pros: $\mathrm{Conf}(m_1, m_1) = 0$
- cons: $\mathrm{Conf}(m_1, m_\Omega) \neq 0$

Digression on distances

Jousselme *et al.* (2001) distance can be done by:

$$d(m_1, m_2) = \sqrt{\frac{1}{2}(\mathbf{m}_1 - \mathbf{m}_2)^T \underline{\underline{D}}(\mathbf{m}_1 - \mathbf{m}_2)},$$

where $\underline{\underline{D}}$ is an $2^{|\Omega|} \times 2^{|\Omega|}$ matrix based on Jaccard dissimilarity

$$D(A,B) = \begin{cases} 1, \text{ if } A = B = \emptyset, \\ \\ \dfrac{|A \cap B|}{|A \cup B|}, \forall A, B \in 2^{\Omega}. \end{cases}$$

Conflict measure: based on inclusion

We define a conflict measure between two mass functions m_1 and m_2 by (Martin, 2012):

$$\mathrm{Conf}(m_1, m_2) = (1 - \delta_{inc}(m_1, m_2))d(m_1, m_2) \tag{9}$$

where d is the distance, δ_{inc} is a degree of inclusion. This measure holds axioms:

1. **Non-negativity:** $\mathrm{Conf}(m_1, m_2) \geq 0$
2. **Identity:** $\mathrm{Conf}(m_1, m_1) = 0$
3. **Symmetry:** $\mathrm{Conf}(m_1, m_2) = \mathrm{Conf}(m_2, m_1)$
4. **Normalization:** $0 \leq \mathrm{Conf}(m_1, m_2) \leq 1$
5. **Inclusion:** $\mathrm{Conf}(m_1, m_2) = 0,$ iff $m_1 \subseteq m_2$ or $m_2 \subseteq m_1$
   The mass functions cannot be in conflict if one is included in the other one.

Reliability estimation

If the conflict comes from unreliable sources: reliability estimation

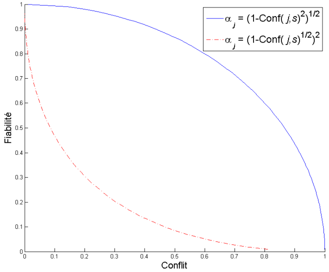
Assumption: a source is unreliable if it is in conflict with the other sources

Reliability measure: such as a decreasing function of the conflict measure

$$\alpha_j = f(\mathrm{Conf}(j,s))$$
$$\alpha_j = \left(1 - \mathrm{Conf}(j,s)^\lambda\right)^{1/\lambda}$$

Integration of the reliability measure by discounting

$$m_j^\alpha(X) = \alpha_j m_j(X),\ \forall X \in 2^\Omega$$
$$m_j^\alpha(\Omega) = 1 - \alpha_j(1 - m_j(\Omega))$$



Zadeh example

	$\emptyset$	ω_1	ω_2	ω_3	Ω	Decision with betP
m_1	0	0.9	0	0.1	0	ω_1
m_2	0	0	0.9	0.1	0	ω_2
m_{D}	0	0	0	1	0	ω_3

Solutions

- Conflict coming from a false assumption of closed world
- Conflict coming from the assumption of source's independence
- Conflict coming from source's ignorance assumption
- Conflict coming from source reliability assumption
- Conflict coming from a number of sources

False assumption of closed world

- Closed world: Frame of discernment assumed to be exhaustive
- Smets interpreted $m(\emptyset) > 0$ such as another element and used the conjunctive rule m_{Conj}
- $m(\emptyset)$ is composed of all the partial conflicts: we can considered such as many unknown elements.

The PCR6 (Martin et Osswald, 2006, 2007)

This rule transfers the partial conflicts on the elements that generate it, proportionally to their mass functions.

$$m_{\mathrm{PCR6}}(X) = m_{\mathrm{Conj}}(X) + \sum_{\substack{Y \in D^\Omega, \\ X \cap Y \equiv \emptyset}} \left(\frac{m_1(X)^2 m_2(Y)}{m_1(X) + m_2(Y)} + \frac{m_2(X)^2 m_1(Y)}{m_2(X) + m_1(Y)} \right) \tag{10}$$

$$m_{\mathrm{PCR6}}(X) \quad = \quad m_{\mathrm{Conj}}(X) + \sum_{j=1}^{s} m_i(X)^2 \sum_{\substack{\bigcap_{j'=1}^{s-1} Y_{\sigma_j(j')} \cap X = \emptyset \\ (Y_{\sigma_j(1)},\ldots,Y_{\sigma_j(s-1)}) \in (D_r^\Omega)^{s-1}}} \left(\frac{\prod_{j'=1}^{s-1} m_{\sigma_j(j')}(Y_{\sigma_j(j')})}{m_j(X) + \sum_{j'=1}^{s-1} m_{\sigma_j(j')}(Y_{\sigma_j(j')})} \right)$$

Assumption of source's independence

- If dependent sources: combination rule has to be idempotent
- The simplest way: the average of the mass functions:

$$m_{\mathrm{M}}(X) = \frac{1}{s}\sum_{j=1}^{S} m_j(Y_j). \qquad (11)$$

- The cautious rule of S separable mass functions $m_j, j = 1, 2, \cdots, S$:

$$m_{\mathrm{cau}}(X) = \bigcirc\!\!\!\!\cap_{A \subsetneq \Omega} A^{\bigwedge_{j=1}^{S} w_j(A)} \qquad (12)$$

where $A^{w_j(A)}$ is the simple support function focused on A with weight function $w_j(A)$ issued from the canonical decomposition of m_j. Note also that $\wedge$ is the min operator.

Source's ignorance

Yager (1987) rule

- Assumption: the global conflict comes from the ignorance
- Stay in closed world
- The mass of the empty set is transfered on the global ignorance Ω

$$
\begin{aligned}
m_{\mathrm{Y}}(X) &= m_{\mathrm{Conj}}(X), \forall X \in 2^{\Omega} \setminus \{\emptyset, \Omega\} \\
m_{\mathrm{Y}}(\Omega) &= m_{\mathrm{Conj}}(\Omega) + m_{\mathrm{Conj}}(\emptyset) \\
m_{\mathrm{Y}}(\emptyset) &= 0.
\end{aligned}
$$

Dubois and Prade (1988) rule

- Assumption: Partial conflict comes from the partial ignorances
- The partial conflict are transfered on the partial ignorances

$$m_{\mathrm{DP}}(X) = \sum_{A \cap B = X} m_1(A) m_2(B) + \sum_{\substack{A \cup B = X \\ A \cap B = \emptyset}} m_1(A) m_2(B).$$

- Precise transfer of the global conflict
- Algorithm complexity higher

Reliability assumption

Disjunctive rule

- If no knowledge about reliability: at least one source is reliable

$$m_{\mathrm{Dis}}(X) = \sum_{Y_1 \cup \ldots \cup Y_s = X} \prod_{j=1}^{S} m_j(Y_j). \tag{13}$$

- Main problem: lost of specificity after combination

Reliability assumption

Florea (2006) rule

- Stay in closed world
- Propose a global conflict transfer in a such way more the global conflict is high more the rule has a disjunctive comportment.
- The rule is given $\forall X \in 2^\Omega$, $X \neq \emptyset$ by:

$$m_{\mathrm{Flo}}(X) = \beta_1(\kappa) m_{\mathrm{Dis}}(X) + \beta_2(\kappa) m_{\mathrm{Conj}}(X),$$

  where β_1 and β_2 have $\kappa = \dfrac{1}{2}$ like a symmetric weight:

$$\beta_1(\kappa) = \frac{\kappa}{1-\kappa+\kappa^2},$$

$$\beta_2(\kappa) = \frac{1-\kappa}{1-\kappa+\kappa^2}.$$

- We have seen that 1/2 cannot be the symmetric value of the global conflict
- Other weights are proposed

(Martin et Osswald, 2007): **Mixed rule**

- General distribution of partial conflict

$$\begin{aligned} m_{\mathrm{Mix}}(X) &= \sum_{Y_1 \cup Y_2 = X} \delta_1 m_1(Y_1) m_2(Y_2) \\ &+ \sum_{Y_1 \cap Y_2 = X} \delta_2 m_1(Y_1) m_2(Y_2) \end{aligned} \tag{14}$$

- Dubois and Prade's rule
$\delta_1(Y_1, Y_2) = 1 - \delta_2(Y_1, Y_2) = \mathbb{1}_{Y_1 \cap Y_2 = \emptyset}(Y_1, Y_2)$
and Florea's rule can be seen such as a particular case

Mixed rule: taking into account the specificity

The choice of $\delta_1 = 1 - \delta_2$ can be given from a dissimilarity measure such as:

$$\delta_1^1(Y_1, Y_2) = 1 - \frac{|Y_1 \cap Y_2|}{\min(|Y_1|, |Y_2|)} \tag{15}$$

or from the Jaccard's dissimilarity:

$$\delta_1^2(Y_1, Y_2) = 1 - \frac{|Y_1 \cap Y_2|}{|Y_1 \cup Y_2|} \tag{16}$$

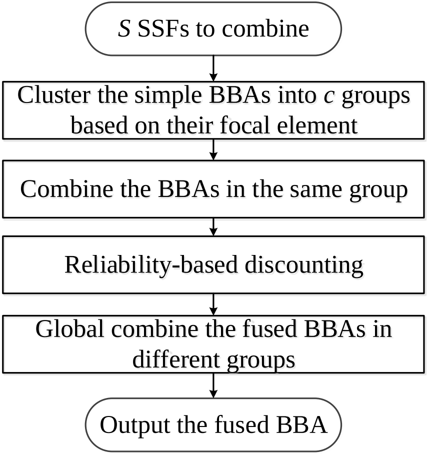
- If $Y_1 \cap Y_2 = \emptyset$: partial conflicts can be interpreted such as partial ignorances
- If $Y_1 \cap Y_2 \notin \{Y_1, Y_2, \emptyset\}$: transfer on $Y_1 \cap Y_2$ and $Y_1 \cup Y_2$ according to δ_1^1 and δ_1^2

LNS rule: (Zhou *et al.* 2017)

- Problems:
  - Many sources: assumption of the reliability of all the sources difficult to consider
  - Disjunctive rule: at least one source reliable but lost of specificity
- Assumptions of LNS rule
  - Majority of sources are reliable
  - The larger extent one source is consistent with others, the more reliable the source is
  - Sources are cognitively independent

Number of sources

LNS rule: (Zhou *et al.* 2017)



LNS rule: (Zhou *et al.* 2017)

For each mass function m_j we consider the set of mass functions $\{A_k^{w_j}, A_k \subset \Omega\}$ coming from the canonical decomposition. If group the simple mass functions $A_k^{w_j}$ in c clusters (the number of distinct A_k) and denote by s_k the number of simple mass functions in the cluster k, the proposed rule is given by:

$$m_{\mathrm{LNS}} = \textcircled{\cap}_{k=1,\cdots,c}(A_k)^{1-\alpha_k+\alpha_k \prod_{j=1}^{s_k} w_j} \tag{17}$$

where

$$\alpha_k = \frac{s_k}{\sum_{i=1}^{c} s_i}. \tag{18}$$

What to do to manage the conflict?

- First, if you have any reliable information on the reliability of the sources: discount the mass functions.
- If there is still global conflict: choose an appropriate rule to manage this conflict.
- There is no an optimal rule for any application.
  1. Choose the rule according to the needed properties (reliability, idempotence, independence, complexity, etc.)
  2. Try some rules and choose the best for your application

Such m_1 and m_2 two mass functions defined on $\Omega = \{\omega_1, \omega_2, \omega_3, \omega_4\}$ by:

$m_1(\omega_1) = 0.3$, $m_1(\omega_1, \omega_3) = 0.5$ and $m_1(\omega_2, \omega_3, \omega_4) = 0.2$

$m_2(\omega_2, \omega_3) = 0.4$, $m_2(\omega_4) = 0.5$ and $m_2(\omega_1, \omega_3, \omega_4) = 0.1$

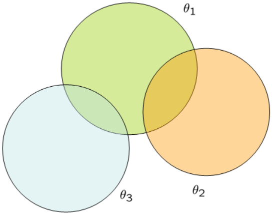
Give the obtained masses by conjunctive combinaison (normalized or not), disjunctive, Yager's combination and Dubois and Prade combination.

Outline

1. Introduction to information fusion
2. Theory of belief functions for classifier fusion
3. Managing conflict
4. Decisions with conflicting bbas

Decisions with conflicting bbas

- Another point of view: do not manage the conflict and keep it until the decision
- That is the point of view of Smets with the use of the conjunctive rule and the pignistic probability for the decision.
- Other idea: keep all the partial conflicts during the combination rule:
  - Hyper power set $D^\Omega = \{\omega_1 \cap \omega_2, \omega_1, \omega_2, \omega_1 \cup \omega_2\}$, if $\Omega = \{\omega_1, \omega_2\}$
  - D_r^Ω: reduced set with constraints $(\omega_2 \cap \omega_3 \equiv \emptyset)$



Decision

- In D^Ω classic decision can be made on Ω without partial conflict
  - Pessimist: $\max_{\omega \in \Omega} bel(\omega)$
  - Optimist: $\max_{\omega \in \Omega} pl(\omega)$
  - Compromise: $\max_{\omega \in \Omega} betP(\omega)$

Pignistic probability:

$$\mathrm{GPT}(X) = \sum_{Y \in D_r^\Omega, Y \not\equiv \emptyset} \frac{\mathcal{C}_\mathcal{M}(X \cap Y)}{\mathcal{C}_\mathcal{M}(Y)} m(Y) \qquad (19)$$

where $\mathcal{C}_\mathcal{M}(X)$ is the cardinality X of D_r^Ω

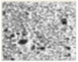
Decision in presence of conflict

Example

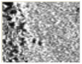
Data GESMA

Sand

Rock



Rock OR Sand



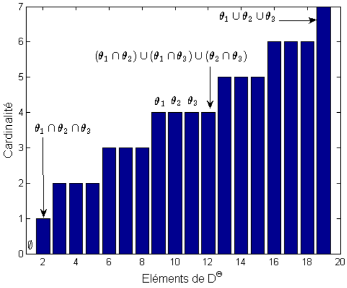
Rock AND Sand

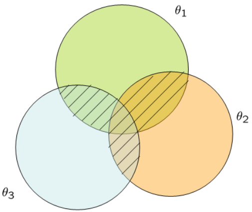
Wreck or object

Decision on conflicting bbas

(Martin, 2008): decision on D^Ω

- on D_r^Ω
  $$A = \operatorname*{argmax}_{X \in D_r^\Omega} \left(m_d(X) f_d(X) \right)$$
  $$m_d(X) = \left(\frac{K_d \lambda_X}{\mathcal{C}_\mathcal{M}(X)^r} \right)$$
- on $\mathcal{D}$ a subset of D^Ω
  $$A = \operatorname*{argmax}_{X \in \mathcal{D}} \left(m_d(X) f_d(X) \right)$$
  such as the cardinality
  $$\mathcal{D} = \left\{ X \in D_r^\Omega ; \min_C \leq \mathcal{C}_\mathcal{M}(X) \leq \max_C \right\}$$





Decision on conflicting bbas

Decision on 2^Ω (Essaid, *et al.* 2014)

$$A = \operatorname*{argmin}_{X \in \mathcal{D}} d(m, m_X),$$

where

- $\mathcal{D}$ is the set of elements of 2^Ω on which we want to decide,
- m_X is a categorical mass function,
- d is a distance on mass functions (such as Jousselme distance)
- m is the mass function coming from the function

No threshold r to fit to decide on imprecise element of 2^Ω

To end

The theory of belief functions provides an appropriate framework for information fusion.

Take care of conflict

- Model the conflict correctly
- Suppress the conflict
- Manage the conflict in the combination rule
- Keep the conflict until the decision step

DRUID

http://www-druid.irisa.fr/

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